3.2 Characteristics of Polynomial Functions

A End Behaviour For $\lvert x \rvert$ very large ($x \to \infty$ or $x \to -\infty$) the graph of the polynomial function resemble the graph of the leading term $y = a_n x^n$.	Ex 1. For each case, find an example and sketch the *end behaviour* of the polynomial function.
1a) $a_n > 0$, *even* n	1b) $a_n > 0$, *odd* n
1c) $a_n < 0$, *even* n	1d) $a_n < 0$, *odd* n
B Symmetry A polynomial function is *even* if $f(-x) = f(x)$. In this case: • all exponents of x are even • graph is symmetric with respect to the y-axis A polynomial function is *odd* if $f(-x) = -f(x)$. In this case: • all exponents of x are odd • graph is symmetric with respect to the origin	Ex 2. Classify as even, odd, or neither. a) $f(x) = x - x^3 + 2x^5$ b) $f(x) = 2 - x^2 + 3x^6$ c) $f(x) = 1 + x - x^3 + 3x^4$ d) $f(x) = (x^2 - 1)^3$ e) $f(x) = -x^3(x^2 + 1)^2$
C Zeros versus x-intercepts A *zero* z is a number (real or complex) where the *value* of the function is zero. So $y = f(z) = 0$. An *x-intercept* $x - \text{int}$ is a *real number* where the graph of the function intersects the x-axis. So $y = f(x - \text{int}) = 0$. Notes. • A real zeros is also an x-intercepts. • A complex zero is not an x-intercept.	Ex 3. (Optional) Find the zeros and the x-intercepts for $f(x) = (x-1)(x^2+1)$

D Fundamental Theorem of Algebra

A polynomial function $y = P(x)$ of degree n has n zeros (real or complex).

Notes:

- If the *coefficients* of the polynomial function are *real numbers* then complex zeros come in conjugate pairs (the number of complex zeros may be 0 (none), 2, 4, 6, $\ldots$)
- The number of *real zeros* is *at most* n. These zeros may be distinct (different) or coincident (same)
- A polynomial function of *even* degree *may have no real zero* (all may be complex).
- A polynomial function of *odd* degree must have *at least one* real zero.

Ex 4. Analyze the general shape for the following polynomial functions.

a) linear

b) quadratic

c) cubic

d) quartic

E Turning Points

A turning point is the point where an increasing behavior changes to decreasing behavior or vice versa.

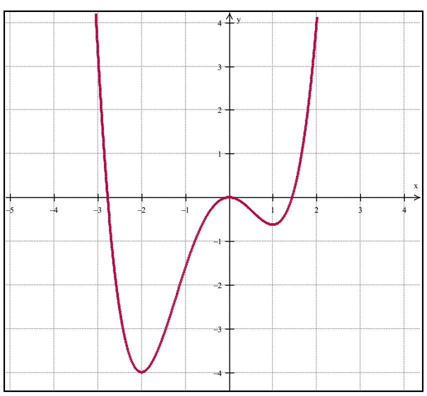
A polynomial function of degree n has at most $n-1$ turning points.

A turning point may be either a *maximum* or a *minimum* point.

F Extrema Points

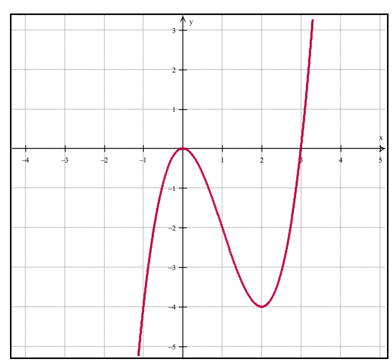
Extrema is the plural of extremum.
Extremum is either a minimum or a maximum.
An extremum may be local (relative) or global (absolute).
Each turning point is a local extremum point.
A polynomial function of even degree has either a global (absolute) minimum or a global (absolute) maximum point.
A polynomial function of odd degree has neither a global (absolute) minimum nor a global (absolute) maximum point.

Ex 5. Find the turning points for the polynomial function represented bellow. What degree may have this polynomial function? Is this degree odd or even?



Ex 6. Find the local (relative) and global (absolute) extremum points for the polynomial function at example 5.

Ex 7. Find the local (relative) and global (absolute) extremum points for the polynomial function represented bellow.



Reading: Nelson Textbook, Pages 129-135
Homework: Nelson Textbook, Page 136: #1, 3, 5, 7, 10, 11, 14, 16

